

# CP violation in the heavy neutrinos production process $e^+e^- \rightarrow N_1 N_2$ \*

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## Abstract

The problem of CP conservation and CP violation for two heavy neutrinos production in  $e^+e^-$  interaction is considered. Very convenient way of parametrization of the neutrino mass matrix, from which necessary and sufficient condition for CP conservation easily follows, is presented. Contrary to the Kobayashi-Maskawa mechanism, the effects of CP violation in the lepton sector with Majorana neutrinos can be very large. Change of the total cross section caused by CP violation can be much larger than the cross section itself.

## 1 Introduction

The origin of CP violation is one of the most important open problems in particle physics. In the standard model (SM) the CP violation is explained by the Kobayashi-Maskawa mechanism [1]. In this mechanism the CP violation depends on mixing

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between flavour eigenstates and mass eigenstates. For the mixing to take place, the fermions with given charges must have distinguishable masses. That is why the CP violation is visible in the quark sector (quark masses are distinguishable) and not visible in the lepton sector (light neutrinos masses are still consistent with zero). The CP violation effect has been observed until now only in  $K^0 - \bar{K}^0$  sector [2] and is small. It is because the only quantity which describes the CP violation in the KM mechanism is the parameter  $\delta_{KM}$  given by

$$\delta_{KM} = \text{Im} (V_{cd}V_{ub}V_{cb}^*V_{ud}^*). \quad (1)$$

As the KM mixing matrix parameters  $V_{ik}$  are small the  $\delta_{KM}$  is also small

$$\delta_{KM} < 10^{-4}. \quad (2)$$

The CP violation problem is very interesting in the lepton sector if the neutrinos are Majorana particles. First of all, contrary to the Dirac particles, the physical Majorana fields are not rephasing invariant. Then not so much phases can be eliminated and CP is violated already for two generations of leptons [3]. The greater number of non-eliminated phase parameters is also the cause why the CP violation is not mass suppressed [4] so the effect could be potentially visible even for very light neutrinos.

In this paper we consider the problem of CP violation in the case of heavy Majorana neutrinos. Such particles with the masses greater than 100 GeV can be produced in the future  $e^+e^-$  colliders. All our considerations are done in the framework of the Left-Right (L-R) symmetric model which predicts the existence of the Majorana neutrino in a natural way. In the next Chapter we find the most convenient parametrization of the mass matrix for the study of CP violation. A necessary and sufficient condition guaranteeing CP invariance on the level of weak lepton states is studied. The numerical analysis of the CP violation in the  $e^+e^- \rightarrow N_1N_2$  process are done in Chapter 3 and some conclusions are presented at the end.

## 2 Parametrization of the mass and mixing matrices

We consider the L-R model [5] described in details in Refs[6]. The relevant parts of the model's lagrangian for studying the CP properties are the charged-current interaction and the lepton mass lagrangian. They are given by

$$L_{CC} = \frac{g}{\sqrt{2}} \left( \bar{\nu}_L \gamma^\mu l_L W_{L\mu}^+ + \bar{\nu}_R \gamma^\mu l_R W_{R\mu}^+ \right) + h.c. \quad (3)$$

and

$$L_{mass} = -\frac{1}{2} (\bar{n}_L^c M_\nu n_R + \bar{n}_R M_\nu^* n_L^c) - (\bar{l}_L M_l l_R + \bar{l}_R M_l^+ l_L) \quad (4)$$

where  $n_R$  is six-dimensional vector of the neutrino fields

$$\begin{aligned} n_R &= \begin{pmatrix} \nu_R^c \\ \nu_R \end{pmatrix}, \quad \nu_R^c = i\gamma^2 \nu_L^*, \\ n_L &= \begin{pmatrix} \nu_L \\ \nu_L^c \end{pmatrix}, \quad \nu_L^c = i\gamma^2 \nu_R^*. \end{aligned} \quad (5)$$

$M_\nu$  and  $M_l$  are  $6 \times 6$  and  $3 \times 3$  mass matrices for neutrinos and charged leptons respectively. We consider the model with the explicit left-right symmetry where the left-handed neutral Higgs triplet does not condensate ( $v_L = 0$ ). Then the mass matrix  $M_\nu$  is given by

$$M_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix} \quad (6)$$

where  $3 \times 3$  matrices  $M_D$  (and also  $M_l$ ) are hermitian and  $M_R$  is symmetric. The most general CP transformation which leaves the gauge interactions (3) invariant is [7]

$$\begin{aligned} l_L &\rightarrow V_L C l_L^*, \quad \nu_L \rightarrow V_L C \nu_L^*, \\ l_R &\rightarrow V_R C l_R^*, \quad \nu_R \rightarrow V_R C \nu_R^*. \end{aligned} \quad (7)$$

where  $V_{L,R}$  are  $3 \times 3$  unitary matrices acting in lepton flavour space and  $C$  is the Dirac charge conjugation matrix. For the full lagrangian to be invariant under (7) the lepton mass matrices  $M_D, M_R$  and  $M_l$  have to satisfy the conditions

$$\begin{aligned} V_L^\dagger M_D V_R &= M_D^*, \\ V_R^T M_R V_R &= M_R^*, \end{aligned} \quad (8)$$

and

$$V_L^\dagger M_l V_R = M_l^*. \quad (9)$$

The relations expressed by Eqs.(8) and (9) are weak-basis independent and constitute necessary and sufficient condition for CP invariance. It means that if for given matrices  $M_D, M_R$  and  $M_l$ , there exist two unitary matrices  $V_L$  and  $V_R$  such that relations (8,9) hold then our model is CP invariant and, on the other hand, if CP is the symmetry of our model then such matrices  $V_L$  and  $V_R$  exist. The most convenient basis

for studying CP symmetry is the weak basis in which charged lepton mass matrix  $M_l$  is real, positive and diagonal

$$M_l = \text{diag}[m_e, m_\mu, m_\tau]. \quad (10)$$

Then for non-degenerate, non-vanishing  $m_e \neq m_\mu \neq m_\tau$  Eq.(8) and (9) imply that matrices  $V_{L,R}$  are diagonal and equal

$$V_L = V_R = \text{diag}[e^{i\delta_1}, e^{i\delta_2}, e^{i\delta_3}]. \quad (11)$$

From Eqs.(8) and (9) follows that the model has CP symmetry if and only if the matrices  $M_D$  and  $M_R$  have the elements

$$\begin{aligned} (M_D)_{ij} &= |(M_D)_{ij}| e^{+\frac{i}{2}(\delta_i - \delta_j)}, \\ (M_R)_{ij} &= |(M_R)_{ij}| e^{-\frac{i}{2}(\delta_i + \delta_j)} \end{aligned} \quad (12)$$

in the basis where  $M_l$  is diagonal. The number of reduced phases ( $\frac{n(n+1)}{2}$  for symmetric  $M_R$  and  $\frac{n(n-1)}{2}$  for hermitian  $M_D$  give totally  $n^2$  phases)

$$n^2 - n \quad (= 6) \quad (13)$$

is the lepton sector number of independent CP violating phases in the considered model (with explicite L-R symmetry and  $v_L = 0$ ).

It is easy to understand why relations (12) are necessary and sufficient conditions for CP invariance. From Eqs.(12) follows that the neutrino mass matrix  $M_\nu$  (Eq.6) is diagonalized by the orthogonal transformation

$$U^T M_\nu U = \text{diag}[|m'_1|, \dots, |m'_6|] \quad (14)$$

and the  $(2n \times 2n)$  unitary matrix  $U$  can be expressed in the form

$$U = \begin{pmatrix} V^* & 0 \\ 0 & V \end{pmatrix} O \eta \quad (15)$$

where

$$V = \text{diag}[e^{i\delta_1/2}, e^{i\delta_2/2}, e^{i\delta_3/2}], \quad (16)$$

$O$  is a real orthogonal  $2n \times 2n$  matrix ( $O^T = O^{-1}$ ) that diagonalizes the real part of  $M_\nu$  matrix after removing the phases  $e^{i\delta_i/2}$ , and  $\eta$  is a diagonal  $(2n \times 2n)$  matrix that ensures that the neutrino masses are positive numbers ( $m_i = |m'_i| \geq 0$ )

$$\eta_{ij} = \delta_{ij} e^{i\frac{\pi}{4}(\text{sign}[m'_i] - 1)}. \quad (17)$$

The CP symmetry is then satisfied if we define the CP parity of Majorana neutrinos [8]

$$\eta_{CP}(i) = i \text{sign}[m'_i]. \quad (18)$$

To find the mixing matrices  $K_{L,R}$  for the left (right) charged current and the neutral currents  $\Omega_{L,R}$  (see Ref.[6] for precise definition) we define

$$U \equiv \begin{pmatrix} U_L^* \\ U_R \end{pmatrix} = \begin{pmatrix} V^* O_L \eta \\ V O_R \eta \end{pmatrix}. \quad (19)$$

Then

$$\begin{aligned} K_L \equiv U_L^\dagger &= \eta O_L^T V^\dagger \\ K_R \equiv U_R^\dagger &= \eta^* O_R^T V^\dagger, \end{aligned} \quad (20)$$

and

$$\begin{aligned} \Omega_L \equiv K_L K_L^\dagger &= \eta O_L^T O_L \eta^*, \\ \Omega_R \equiv K_R K_R^\dagger &= \eta^* O_R^T O_R \eta, \\ \Omega_{RL} \equiv K_R K_L^\dagger &= \eta^* O_R^T O_L \eta^*. \end{aligned} \quad (21)$$

From Eqs.(20) and (21) we see that the phase factors from matrix V multiply the columns of the matrices  $K_{L,R}$  and can be absorbed by rephasing of the charged-lepton fields in the charged currents  $l_{L,Ri} \rightarrow e^{i\delta_i/2} l_{L,Ri}$ . The phase factors disappear from matrices  $\Omega_{L,R}$  and  $\Omega_{RL}$  which mix the physical Majorana neutrino fields for which the rephasing is not possible.<sup>1</sup> Then, if the CP is not spontaneously broken, the total lepton lagrangian (gauge-gauge, gauge-leptons, Higgs-leptons and Higgs interactions) is CP invariant. If the phases of matrices  $M_D$  and  $M_R$  differ from those that are given by Eqs.(12) the CP symmetry is broken. In the next Chapter we investigate the effect of these CP broken phases in the production process of two heavy neutrinos.

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<sup>1</sup>We adopt the definition of the physical Majorana fields  $N(x)$  as fields that under charge conjugation stay the same without any phase factor

$$N^C(x) \equiv C \bar{N}^T(x) = N(x).$$

For definition of Majorana fields where the creation phase factors are introduced see Refs.[9]. We do not think that these definitions are useful.

### 3 The CP effect in the process $e^+e^- \rightarrow N_1N_2$ ; numerical analysis.

The amplitude for two Majorana neutrino production process in  $e^+e^-$  interaction is given by the contributions from six diagrams with gauge boson exchange in t,u and s channels (see Fig.1). The contributions from Higgs exchange particles are negligible [10] and we do not consider them here.

Full helicity amplitudes  $M(\sigma\bar{\sigma}; \lambda_1\lambda_2)$  for the process

$$e^-(\sigma) + e^+(\bar{\sigma}) \rightarrow N_1(\lambda_1) + N_2(\lambda_2) \quad (22)$$

are presented in Appendix of Refs[6] and [10].

The CP effects are caused by phase factors that appear in the mixing matrices  $K_{L,R}$  in t and u channels and  $\Omega_{L,R}$  in s channel. To observe the influence of these phases two things must happen. First, different CP phases have to contribute to various Feynman diagrams from Fig.1, and second, the diagrams have to interfere so that at least two Feynman diagrams must contribute to the same helicity amplitude. The same mixing matrix elements give contributions to the  $W_1$ ,  $W_2$  exchange diagrams in t-u channels ( $K_{L,R}$ ) and  $Z_1, Z_2$  bosons exchange in s channel ( $\Omega_{L,R}$ ). So even if these diagrams contribute to the same helicity amplitude they do not interfere (of course there are also other suppression factors as the gauge boson mixing angles are small [6]). If the energy is large compared to the masses of neutrinos  $N_1$  and  $N_2$  then the t channel contributes to  $M(-+; -+)$  (left-handed current) and  $M(+--; +-)$  (right-handed current) and the u channel gives contributions to  $M(-+; +-)$  and  $M(+--; -+)$  amplitudes. We can see that at high energy there is no interference between t and u channels [4]. The s-channel diagrams produce all four helicity amplitudes. So at high energy we can look for CP effects resulting from the interference between t-s and u-s channels.

For the energy just above the production threshold there is no helicity suppression mechanism and final neutrinos with all helicity states can be produced by each channel diagram. These are the best conditions for observing the CP violation effects.

Another question is in what experimental observables the CP effects are visible. From the discussion presented above we can see that they can be looked for in polarized angular distribution. Unfortunately the cross sections, as we shall see, are too small to realize this possibility. And what about the unpolarized angular distribution? If CP is conserved then the helicity amplitude satisfies the relation ( $\Theta$  and  $\phi$  are CM scattering angles)

$$M(\sigma, \bar{\sigma}; \lambda_1, \lambda_2; \Theta, \phi) = -\eta_{CP}^*(1)\eta_{CP}^*(2) \times M(-\bar{\sigma}, -\sigma; -\lambda_1, -\lambda_2; \pi - \Theta, \pi + \phi). \quad (23)$$

where  $\eta_{CP}(i)$  are CP parities of the Majorana neutrinos. If we sum over all helicity the unpolarized angular distribution has forward-backward isotropy

$$\frac{d\sigma}{d\Omega}(\Theta, \phi) = \frac{d\sigma}{d\Omega}(\pi - \Theta, \pi + \phi). \quad (24)$$

Does it mean that anisotropy can be observed if CP is violated? Unfortunately not, at least if we neglect the final state interaction. Without final state interaction from CPT symmetry we can prove the relation

$$M(\sigma, \bar{\sigma}; \lambda_1, \lambda_2; \Theta, \phi) = -\eta_{CP}(1)\eta_{CP}(2)e^{2i(\sigma-\bar{\sigma})(\pi+\phi)} M^*(-\bar{\sigma}, -\sigma; -\lambda_1, -\lambda_2; \pi - \Theta, \pi + \phi) \quad (25)$$

from which the forward-backward isotropy also follows [11]. So the only observables where we can try to find the CP violation effect are the total cross sections. How big the effects can be? There are six phases which cause the CP symmetry breaking. We do not try to find the phase for which the effects of CP breaking is maximal. We take the matrices  $M_D$  and  $M_R$  in the form

$$M_D = \begin{pmatrix} 1. & 1. & .9 \\ 1. & 1. & .9 \\ .9 & .9 & .95 \end{pmatrix},$$

and

$$M_R = \begin{pmatrix} 150e^{i\alpha} & 10 & 20 \\ 10 & 200e^{i\beta} & 10 \\ 20 & 10 & 10^6e^{i\gamma} \end{pmatrix},$$

which produce a reasonable spectrum of light neutrinos. If we compare these matrices with Eq.(12) we see that if only one or more phases ( $\alpha, \beta$  or  $\gamma$ ) are not equal 0 or  $\pi$  the CP is violated. Two heavy neutrinos with masses  $M_1 \simeq 150$  GeV and  $M_2 \simeq 200$  GeV, almost independent of the phases  $\alpha, \beta$  and  $\gamma$ , result from our mass matrix. We calculate the cross section for production of these neutrinos in  $e^+e^-$  scattering

$$e^+e^- \rightarrow N_1(150)N_2(200).$$

The appropriate mixing matrix elements  $(K_{L,R})_{1e}$ ,  $(K_{L,R})_{2e}$  and  $(\Omega_{L,R})_{12}$  depend on the phases  $\alpha$  and  $\beta$  and are almost independent of the phase  $\gamma$ . For  $\alpha = \beta = \gamma = 0$  two neutrinos have equal CP parity and CP is conserved

$$\eta_{CP}(N_1) = \eta_{CP}(N_2) = +i. \quad (26)$$

For  $\alpha = \pi, \beta = \gamma = 0$  CP is also conserved if we introduce the CP parities

$$-\eta_{CP}(N_1) = \eta_{CP}(N_2) = +i. \quad (27)$$

For any other values of phases CP is violated. The production cross sections as energy functions are presented in Fig.2. Two factors affect the behaviour of the cross section. First, there is real CP effect which causes the different interference between various diagrams. Second, for different phases different mixing matrix elements are obtained. In Fig.2 both these effects are taken into account. To find the influence of CP interference only we present in Fig.3 the cross sections for the same mixing matrix elements but with all phases the same as in Fig.2. We can see that the influence of the CP interference is very large. The cross section for production of two neutrinos with opposite CP parity can be several times bigger then the cross section for production of the same CP parity neutrinos. The cross sections for the real CP breaking case are placed between two CP conserving situations. We would like to stress that now the CP effect can be quite large contrary to the Kobayashi-Maskawa mechanism in the quark sector. In the lepton sector with Majorana neutrinos the changes in cross section which result from CP breaking can be several times bigger than the cross section itself. Unfortunately, the calculated cross sections are of the range of several femtobarns so the actual observation of the process for reasonable luminosity will be difficult.

## 4 Conclusions

If Majorana neutrino are present in lepton sector the CP violation effect can be very strong. For two heavy neutrinos production process  $e^+e^- \rightarrow N_1N_2$  the CP violation signals appear as an effect of t-u channel interference just above the treshold and t-s, u-s channels interference for higher energy. The angular distribution for unpolarized  $e^+e^-$  beams and without the measurement of the final neutrinos polarization has forward-backward symmetry even if CP is violated but the final state interaction may be neglected. The total cross section is the quantity which changes dramatically



with various CP violating parameters. Even if the change of total cross section is large the cross section is small what makes the observation of this effect difficult.

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## Figure Captions

**Fig.1** Diagrams with gauge boson exchange which describe the process  $e^-e^+ \rightarrow N_1N_2$  in the left-right symmetric model on the tree level.

**Fig.2** CP and mixing matrix effects for the  $e^-e^+ \rightarrow N_1(150)N_2(200)$  production. Solid line is for  $\alpha = \beta = \gamma = 0$ , dotted line is for  $\alpha = 2.0, \beta = \gamma = 0$  and the third line (solid with asterisks) is for  $\alpha = \pi, \beta = \gamma = 0$  phases. The other L-R model parameters which we used are the following:  $M_{W_2} = 1500$  GeV,  $\beta = \frac{M_{W_1}^2}{M_{W_2}^2}$ ,  $M_{Z_2}^2 = \frac{2\cos^4\Theta_W M_{Z_1}^2}{\cos 2\Theta_W \beta}$ ,  $\xi = \beta$ ,  $\phi = -\frac{(\cos 2\Theta_W)^{3/2}}{2\cos^4\Theta_W}\beta$  (see Ref.[6]).

**Fig.3** The effect of CP violation only on the  $e^-e^+ \rightarrow N_1(150)N_2(200)$  production. Absolute values of mixing matrix elements are the same as the ones for solid line in Fig.2 ( $(K_L)_{1e} = .00535, (K_R)_{1e} = .9819, (K_L)_{2e} = .0058, (K_R)_{2e} = .189, (\Omega_L)_{12} = -(\Omega_R)_{12} = .00009$ ). Solid (dotted) line is for opposite (the same) CP parity of neutrinos (Eqs.27 and 26). Solid line with asterisks is for  $\alpha = 2.0, \beta = \gamma = 0$ , the same as in Fig.2.

Fig.1. Feynman diagrams with gauge boson particles

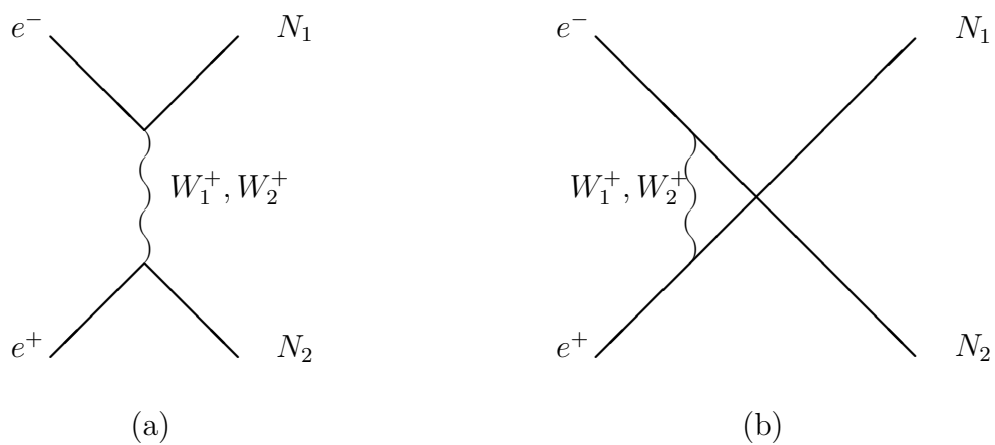


Fig.2

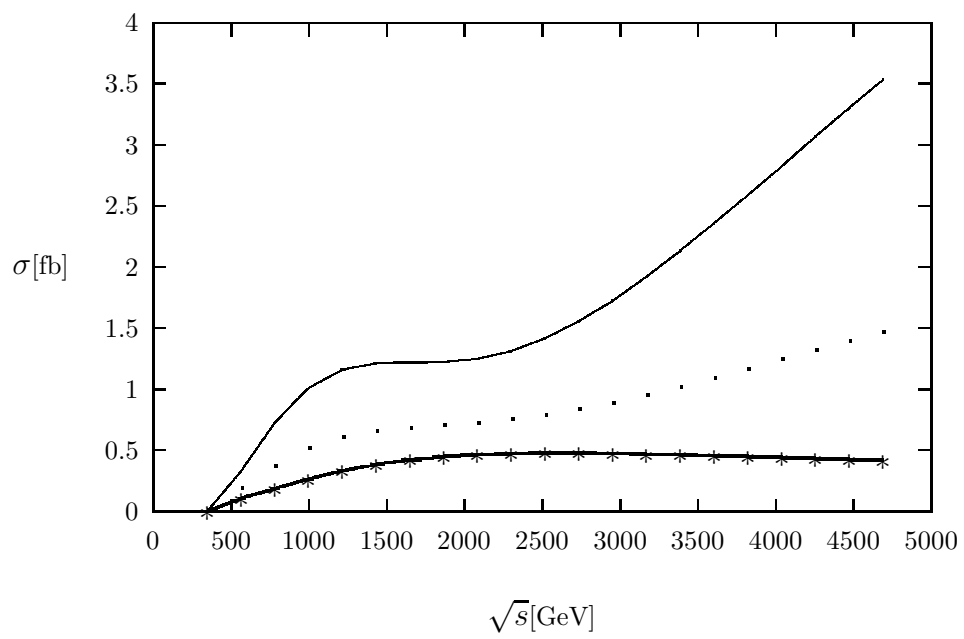


Fig.3

